Exercise 40

For the following exercises, solve the equations over the complex numbers.

$$5x^2 - 8x + 5 = 0$$

Solution

Factor the coefficient of x^2.

$$5\left(x^2 - \frac{8}{5}x + 1\right) = 0$$

The two terms with x, x^2 and $(8/5)x$, cannot be combined, so it's necessary to complete the square to solve for x. Recall the following algebraic identity.

$$(x + B)^2 = x^2 + 2xB + B^2$$

Notice that $2B = -\frac{8}{5}$, which means $B = -\frac{4}{5}$ and $B^2 = \frac{16}{25}$. Add and subtract $\frac{16}{25}$ within the parentheses on the left side and apply the identity.

$$5\left[\left(x^2 - \frac{8}{5}x + \frac{16}{25}\right) + 1 - \frac{16}{25}\right] = 0$$

$$5\left[\left(x + \left(-\frac{4}{5}\right)\right)^2 + \frac{9}{25}\right] = 0$$

$$5\left(x - \frac{4}{5}\right)^2 + \frac{9}{5} = 0$$

Now that x appears in only one place, it can be solved for. Subtract 9/5 from both sides.

$$5\left(x - \frac{4}{5}\right)^2 = -\frac{9}{5}$$

Divide both sides by 5.

$$\left(x - \frac{4}{5}\right)^2 = -\frac{9}{25}$$

Take the square root of both sides.

$$\sqrt{\left(x - \frac{4}{5}\right)^2} = \sqrt{-\frac{9}{25}}$$

$$= \sqrt{\frac{9}{25}(-1)}$$

$$= \sqrt{\frac{9}{25}}\sqrt{-1}$$

$$= \frac{3}{5}i$$

Since there's an even power under an even root, and the result is to an odd power, an absolute value sign is needed around $x - \frac{4}{5}$.

$$\left|x - \frac{4}{5}\right| = \frac{3}{5}i$$

Remove the absolute value sign by placing $\pm$ on the right side.

$$x - \frac{4}{5} = \pm\frac{3}{5}i$$

Add $\frac{4}{5}$ to both sides.

$$x = \frac{4}{5} \pm \frac{3}{5}i$$

Therefore,

$$x = \left\{\frac{4}{5} - \frac{3}{5}i, \frac{4}{5} + \frac{3}{5}i\right\}.$$